General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have:
For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x+3)(x-1)$$

(a) Sketch on the same axes the line L and the curve C, showing the values of the intercepts on the x-axis and the y-axis. *(5 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

(a) xy ; *(1 mark)*

(b) $\dfrac{y}{x}$; *(2 marks)*

(c) $(x+y)^2$. *(3 marks)*

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\dfrac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

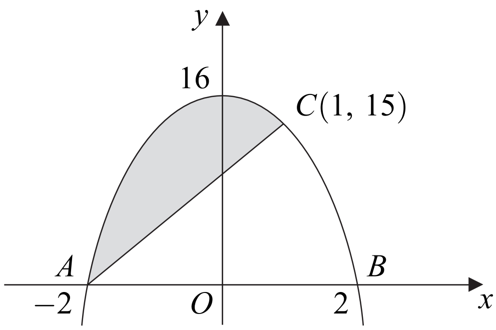
(iii) Find the maximum value of V. *(1 mark)*

4 (a) Express $x^2 - 3x + 4$ in the form $(x - p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)*

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\, dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

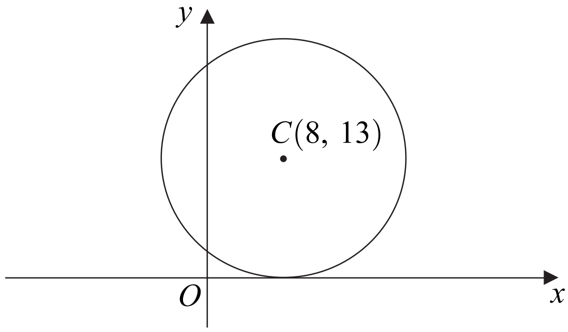
(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

Turn over for the next question

7 The circle S has centre $C(8, 13)$ and touches the x-axis, as shown in the diagram.



(a) Write down an equation for S, giving your answer in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(2 marks)*

(b) The point P with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through P and C. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle S at the point P, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(4 marks)*

(iii) The point Q also lies on the circle S, and the length of PQ is 10. Calculate the shortest distance from C to the chord PQ. *(3 marks)*

8 The quadratic equation $(k+1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS